Skew cellular algebras

and application to Hecke and related algebras of complex reflection groups

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(joint work with Jun Hu and Andrew Mathas)

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Application of Hecke and related algebras
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Cellular algebras (Graham–Lehrer 96)

Let F be a field and A be a finite dimensional F-algebra. The algebra A is cellular if there exists a poset $(\mathcal{P}, \rhd)$ with, for each $\lambda \in \mathcal{P}$,

- an indexing set $\mathcal{T}(\lambda)$,
- elements $c_{\mathfrak{s},\mathfrak{t}} \in A$ for $\mathfrak{s}, \mathfrak{t} \in \mathcal{T}(\lambda)$,

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Use in the study of the representations of A

Cellularity allows to have a combinatorial description of a complete family of non-isomorphic irreducible modules, indexed by a certain subfamily of $\mathcal{P}$.

Examples

Toy example ("triangular fashion")

The algebra $F[x]/(x^n)$ is cellular with the following data:

- $\mathcal{P} := \{0, \dots, n-1\}$ with $\mathcal{T}(i) := \{i\}$,
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The algebra $\mathrm{Mat}_{n\times n}(F)$ is cellular with the following data:

- $\mathcal{P} := \{n\}$ a singleton with $\mathcal{T}(n) := \{1, \dots, n\}$,
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In particular, any semisimple algebra is cellular.

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In particular, any semisimple algebra is cellular.

A non-trivial example of a cellular algebra is the Temperley–Lieb algebra, with poset the set of integers $t \in \{0, \dots, n\}$ satisfying $n - t \in 2\mathbb{Z}$ (Graham–Lehrer 96).

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Complex reflection groups

Definition

- A complex reflection is a linear automorphism of $\mathbb{C}^n$ of finite order, different from identity, that fixes a hyperplane.
- A complex reflection group is a finite subgroup of $\mathrm{GL}(\mathbb{C}^n)$ spanned by complex reflections.

Theorem (Shephard–Todd 54)

Irreducible complex reflection groups are divided into two families:

- *an infinite family* $\{G(r, p, n)\}$ *with* $p \mid r$*;*
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- The group $G(r, p, n)$ is a subgroup of index p of $G(r, 1, n)$.
- To any complex reflection group one can associate a Hecke algebra (Broué-Malle-Rouquier 98).

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$$\begin{aligned} A_{n-1} &= G(1,1,n), & B_n &= G(2,1,n), \\ D_n &= G(2,2,n), & I_2(r) &= G(r,r,2). \end{aligned}$$

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Theorem (Geck 07)

The Hecke algebra of a real reflection group is cellular.

Geck's proof relies heavily on Kazhdan–Lusztig theory.

Ariki–Koike algebras

Definition (Broué–Malle 93, Ariki–Koike 94, BMR 98)

The Ariki–Koike algebra $\mathcal{H}_{r,n}$ is the Hecke algebra of $G(r, 1, n)$. It is isomorphic to a certain "cyclotomic" quotient of the affine Hecke algebra of type A.

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- From Murphy to Hu–Mathas, the order is always the dominance order on r-partitions.
- Webster and Bowman gave in fact many different cellular bases, associated with many different orders (generalising the dominance order). They intensively use a diagrammatic Cherednik algebra.

Temperley–Lieb algebra of type $G(r,1,n)$

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The Temperley–Lieb algebra $\mathcal{TL}_{r,n}$ of type $G(r,1,n)$ is a certain quotient of the Ariki–Koike algebra $\mathcal{H}_{r,n}$.

- $\mathcal{TL}_{1,n}$ is the Temperley–Lieb algebra of type A (Steinberg 82).
- $\mathcal{TL}_{2,n}$ is the Temperley–Lieb algebra of type B or "blob algebra" (defined by Martin–Saleur 94).

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Theorem (Lehrer–Lyu 22)

Assume $r \geq 2$. The algebra $\mathcal{TL}_{r,n}$ is cellular, with poset the set of r-one-column partitions of n with at most two non-empty components.

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Setting

Let A be a cellular algebra and let σ be an algebra automorphism of A.

Question

What can we say about the subalgebra

$$A^\sigma = \{a \in A : \sigma(a) = a\},$$

for instance, is A^σ cellular?

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- We will introduce the notion of skew cellular algebra.
- If σ satisfies some conditions then A^σ will be skew cellular.

Skew cellular algebras (Hu-Mathas-R. 21)

Let A be a finite dimensional F-algebra. The algebra A is cellular if there exists a poset $(\mathcal{P}, \rhd)$ with:

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- a bijection $\iota_\lambda : \mathcal{T}(\lambda) \to \mathcal{T}(\iota\lambda)$ such that $\iota_{\iota\lambda} \circ \iota_\lambda = \mathrm{id}_{\mathcal{T}(\lambda)}$,

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- If $\iota = \mathrm{id}_{\mathcal{P}}$ and $\iota_\lambda = \mathrm{id}_{\mathcal{T}(\lambda)}$ for all $\lambda \in \mathcal{P}$ then we recover Graham–Lehrer's definition of a cellular algebra.

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- We have the same main consequences for the representation theory of A as for cellularity.

Shift automorphisms

Definition (Hu-Mathas-R. 21)

Let $(A, \mathcal{P}, \rhd)$ be a cellular algebra. A shift automorphism of A is a triple $(\sigma_A, \sigma_{\mathcal{P}}, \sigma_{\mathcal{T}})$ where:

- σ_A is an algebra automorphism of A,
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We denote by $\mathcal{P}_\sigma$ the poset of orbits of $\mathcal{P}$ under the action of $\langle \sigma_\mathcal{P} \rangle$.

Skew cellularity of the subalgebra of fixed points

Theorem (Hu-Mathas-R. 21)

Let A be a cellular algebra with a shift automorphism $(\sigma_A, \sigma_{\mathcal{P}}, \sigma_{\mathcal{T}})$. Assume that F contains a primitive p-th root of unity, where p is the order of σ_A. Then the subalgebra A^σ of fixed points is a skew cellular algebra.

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In more details:

- the poset is $\{(\Lambda, k) : \Lambda \in \mathcal{P}_\sigma, k \in \mathbb{Z}/\mathrm{o}_\Lambda\mathbb{Z}\}$ where $\mathrm{o}_\Lambda \mid p$,
- the involution ι on the poset is $(\Lambda, k) \longmapsto (\Lambda, -k)$,
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Corollary

If σ_A has order $p = 2$ then A^σ is cellular.

You are here

Skew cellularity of the Hecke algebra of $G(r, p, n)$

Under some assumptions, the Ariki–Koike algebra $\mathcal{H}_{r,n}$ is naturally equipped with an automorphism σ of order $p \mid r$.

Proposition (Ariki 95)

The Hecke algebra $\mathcal{H}_{r,p,n}$ of $G(r, p, n)$ is isomorphic to the subalgebra of fixed points of $\mathcal{H}_{r,n}$ for the automorphism σ.

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For a particular cellular structure on $\mathcal{H}_{r,n}$, the automorphism σ is a shift automorphism. In particular:

- *the Hecke algebra $\mathcal{H}_{r,p,n}$ of $G(r, p, n)$ is skew cellular,*
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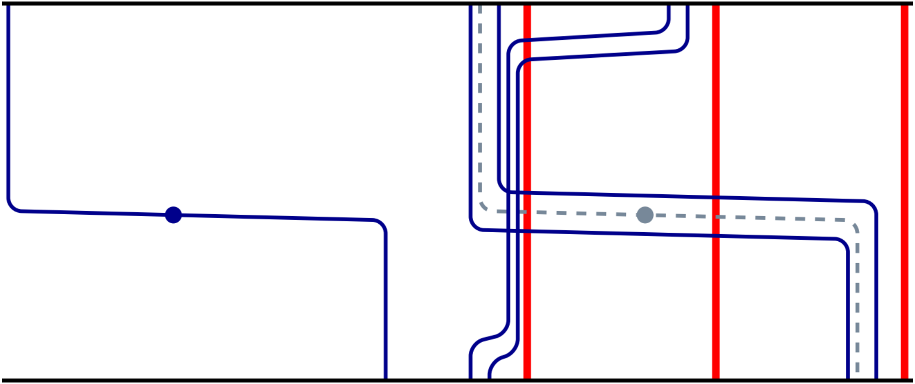
- An important point of the proof is to construct very explicit diagrams inside the Cherednik algebra of Webster–Bowman.
- Geck's result can be applied for $(r, p, n) \in \{(2, 2, n), (p, p, 2)\}$. Our result is stronger that Geck's in the $(2, 2, n)$ case since we prove in general the graded skew cellularity.

A glimpse inside the diagrammatic Cherednik algebra

Here is an example of a kind of diagram, inside the diagrammatic Cherednik algebra of Webster–Bowman, that we used to prove the previous theorem:



Cellularity of the Temperley–Lieb algebra of type $G(r, p, n)$

Under some assumptions, the previous automorphism σ of $\mathcal{H}_{r,n}$ induces an automorphism of the Temperley–Lieb algebra $\mathcal{TL}_{r,n}$ of type $G(r, 1, n)$.

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Theorem (Lehrer–Lyu 22)

The Temperley–Lieb algebra of type $G(r, p, n)$ is cellular.

In fact Lehrer–Lyu prove the skew cellularity, and it turns out that the associated maps ι are the identity maps, whence the cellularity.

The end

Thank you!