A quiver Hecke-like presentation for the Hecke algebra of $G(r, p, n)$

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Ariki–Koike algebra

Let $n, e, p \in \mathbb{N}^*$ with $e \geq 2$. Let q, ζ be some elements of a field F of respective order e, p. Let $\mathbf{\Lambda} = (\Lambda_i)_i$ be a $\mathbb{Z}/e\mathbb{Z}$-tuple of non-negative integers and set $r := p \sum_i \Lambda_i$.

The *Ariki–Koike algebra* $\mathrm{H}_n^{\mathbf{\Lambda}}(q)$ is a Hecke algebra of the complex reflection group $G(r, 1, n)$. It is a F-algebra generated by $S, T_1, \dots, T_{n-1}$, the "cyclotomic relation" being:

$$\prod_{i \in \mathbb{Z}/e\mathbb{Z}} \prod_{j \in \mathbb{Z}/p\mathbb{Z}} (S - \zeta^j q^i)^{\Lambda_i} = \prod_{i \in \mathbb{Z}/e\mathbb{Z}} (S^p - q^{pi})^{\Lambda_i} = 0.$$

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Remarks

- $G(r, 1, n) \simeq (\mathbb{Z}/r\mathbb{Z})^n \rtimes \mathfrak{S}_n$.
- The algebra $\mathrm{H}_n^{\boldsymbol{\Lambda}}(q)$ is a deformation of the group algebra $F[G(r, 1, n)]$.

Basis for the Ariki–Koike algebra

Definition

We set $X_1 := S$ and for $a \in \{1, \ldots, n-1\}$ we define $X_{a+1} \in \mathrm{H}_n^{\Lambda}(q)$ by:

$$qX_{a+1} := T_a X_a T_a.$$

For $a \in \{1, \ldots, n-1\}$, we denote by s_a the transposition $(a, a+1) \in \mathfrak{S}_n$. Let $w \in \mathfrak{S}_n$ and let ℓ minimal such that there exist $a_1, \ldots, a_\ell \in \{1, \ldots, n-1\}$ with $w = s_{a_1} \cdots s_{a_\ell}$. We now define:

$$T_w := T_{a_1} \cdots T_{a_\ell} \in \mathrm{H}_n^{\Lambda}(q).$$

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This element T_w depends only on w and not on $a_1, \dots, a_\ell$.

Proposition

The elements $X_1^{m_1} \cdots X_n^{m_n} T_w$ for $m_a \in \{0, \dots, r-1\}$ and $w \in \mathfrak{S}_n$ form an F-basis of $\mathrm{H}_n^{\Lambda}(q)$.

Cyclotomic quiver Hecke algebra

Let Γ be a quiver ($=$ oriented graph) with vertex set K. The *quiver Hecke algebra* $\mathrm{R}_n(\Gamma)$ is generated over F by:

$$e(\boldsymbol{k}) \text{ for } \boldsymbol{k} \in K^n,$$
$$y_1, \dots, y_n,$$
$$\psi_1, \dots, \psi_{n-1},$$

together with some relations. This algebra has a natural $\mathbb{Z}$-grading.

Exemple of relation

For $\boldsymbol{k} \in K^n$ such that $k_a \xrightarrow{\Gamma} k_{a+1}$ then $\psi_a^2 e(\boldsymbol{k}) = (y_{a+1} - y_a) e(\boldsymbol{k})$.

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For $\boldsymbol{\Lambda} = (\Lambda_k)_{k \in K} \in \mathbb{N}^K$, the *cyclotomic* quiver Hecke algebra $\mathrm{R}_n^{\boldsymbol{\Lambda}}(\Gamma)$ is the quotient of $\mathrm{R}_n(\Gamma)$ by the relations $y_1^{\Lambda_{k_1}} e(\boldsymbol{k}) = 0$ for $\boldsymbol{k} \in K^n$.

Basis for the quiver Hecke algebra

Similarly to the definition of T_w, let $w \in \mathfrak{S}_n$ and let ℓ minimal such that $w = s_{a_1} \cdots s_{a_\ell}$ with $a_1, \ldots, a_\ell \in \{1, \ldots, n-1\}$. We can define:

$$\psi_w := \psi_{a_1} \cdots \psi_{a_\ell} \in \mathrm{R}_n(\Gamma).$$

Remark

Contrary to T_w, the element ψ_w may depend on the chosen $a_1, \ldots, a_\ell$.

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Proposition

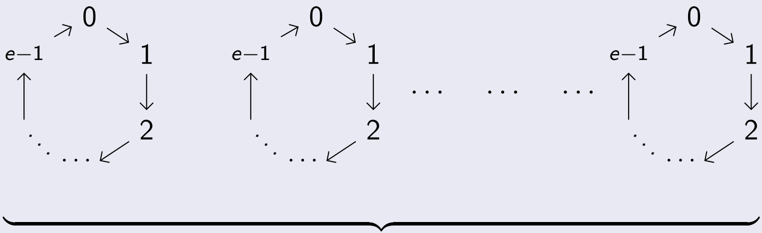
The elements $y_1^{a_1} \cdots y_n^{a_n} \psi_w e(\boldsymbol{k})$ for $a_i \in \mathbb{N}, w \in \mathfrak{S}_n$ and $\boldsymbol{k} \in K^n$ form an F-basis of $\mathrm{R}_n(\Gamma)$.

The image of this basis in the cyclotomic quotient $\mathrm{R}_n(\Gamma)$ spans $\mathrm{R}_n(\Gamma)$ over F, but it is not clear at all how to extract a basis.

Graded isomorphism theorem (I)

Theorem (Brundan–Kleshchev, Rouquier, 08)

The Ariki–Koike algebra $\mathrm{H}_n^{\Lambda}(q)$ *is isomorphic over* F *to the cyclotomic quiver Hecke algebra* $\mathrm{R}_n^{\Lambda}(\Gamma_{e,p})$, *where* $\Gamma_{e,p}$ *is given by:*



$p' := \frac{p}{\gcd(p,e)}$ *copies*

Graded isomorphism theorem (II)

- The set of vertices of $\Gamma_{e,p}$ is :

$$\left\{\zeta^j q^i : i \in \mathbb{Z}/e\mathbb{Z}, j \in \mathbb{Z}/p\mathbb{Z}\right\}.$$

In particular, the integer p' is the smallest $m \geq 1$ such that $\zeta^m \in \langle q \rangle$: hence, the vertex set of $\Gamma_{e,p}$ is in bijection with $\mathbb{Z}/e\mathbb{Z} \times \mathbb{Z}/p'\mathbb{Z}$.

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Corollary 1

The Ariki–Koike algebra is (non-trivially) $\mathbb{Z}$-graded.

Corollary 2

If $\widetilde{q}$ is another primitive eth root of unity in F then $\mathrm{H}_n^{\Lambda}(q) \simeq \mathrm{H}_n^{\Lambda}(\widetilde{q})$.

Graded isomorphism theorem (III)

The isomorphism of Brundan and Kleshchev isomorphism is constructed using some elements $P_a(\boldsymbol{i}, \boldsymbol{j})$ and $Q_a(\boldsymbol{i}, \boldsymbol{j})$ of $F[[y_a, y_{a+1}]]$, for $a \in \{1, \ldots, n-1\}$, $\boldsymbol{i} \in (\mathbb{Z}/e\mathbb{Z})^n$ and $\boldsymbol{j} \in (\mathbb{Z}/p'\mathbb{Z})^n$. These elements have to satisfy some properties depending of $\Gamma_{e,p}$.

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$$X_a \mapsto \sum_{\boldsymbol{i},\boldsymbol{j}} \zeta^{j_a} q^{i_a} (1 - y_a) e(\boldsymbol{i},\boldsymbol{j}),$$

for $1 \leq a \leq n$ and:

$$T_a \mapsto \sum_{\boldsymbol{i},\boldsymbol{j}} \left[\psi_a Q_a(\boldsymbol{i},\boldsymbol{j}) - P_a(\boldsymbol{i},\boldsymbol{j})\right] e(\boldsymbol{i},\boldsymbol{j}),$$

for $1 \leq a \leq n-1$.

Analogue for the Hecke algebra of $G(r, p, n)$

We define an automorphism σ of order p of $\mathrm{H}_n^{\mathbf{\Lambda}}(q)$ by setting:

$$\sigma(S) := \zeta S, \qquad \forall a, \sigma(T_a) := T_a.$$

The subalgebra $\mathrm{H}_{p,n}^{\mathbf{\Lambda}}(q) := \mathrm{H}_n^{\mathbf{\Lambda}}(q)^{\sigma}$ of fixed points is a Hecke algebra of the complex reflection group $G(r, p, n)$.

Remark

The subalgebra $\mathrm{H}_{p,n}^{\mathbf{\Lambda}}(q)$ does not depend on the choice of ζ.

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The subalgebra $\mathrm{H}_{p,n}^{\Lambda}(q)$ does not depend on the choice of ζ.

Question

Can we transpose the previous results involving the Ariki–Koike algebra to the subalgebra $\mathrm{H}_{p,n}^{\Lambda}(q) \subseteq \mathrm{H}_n^{\Lambda}(q)$?

In particular:

- is it isomorphism to something like a cyclotomic quiver Hecke algebra;
- is it a graded subalgebra;
- does it depend on the choosen roots of unity.

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Fixed-point quiver Hecke subalgebra

Let $\sigma : K \to K$ a bijection of finite order p such that:

$$\forall k, k' \in K, k \to k' \implies \sigma(k) \to \sigma(k').$$

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Proposition

The map σ induces a well-defined homogeneous automorphism of $\mathrm{R}_n(\Gamma)$ by:

$$\begin{aligned} \forall \boldsymbol{k} \in K^n, &\quad \sigma(e(\boldsymbol{k})) \coloneqq e(\sigma(\boldsymbol{k})), \\ \forall a \in \{1, \dots, n\}, &\quad \sigma(y_a) \coloneqq y_a, \\ \forall a \in \{1, \dots, n-1\}, &\quad \sigma(\psi_a) \coloneqq \psi_a. \end{aligned}$$

Definition

We set:

$$\mathrm{R}_n(\Gamma)^\sigma \coloneqq \{h \in \mathrm{R}_n(\Gamma) : \sigma(h) = h\}.$$

Fixed-point quiver Hecke subalgebra

Theorem (R., 2016)

We can give a (nice) presentation of $\mathrm{R}_n(\Gamma)^\sigma$ *in terms of the following generators:*

$$e(\gamma) := e(\boldsymbol{k}) + e(\sigma(\boldsymbol{k})) + \cdots + e(\sigma^{p-1}(\boldsymbol{k})) \text{ for } \gamma = [\boldsymbol{k}] \in K^n/\langle\sigma\rangle,$$

$$y_1, \ldots, y_n,$$

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Exemple of relation

If $\gamma \in K^n/\langle\sigma\rangle$ verifies "$\gamma_a \to \gamma_{a+1}$" then $\psi_a^2 e(\gamma) = (y_{a+1} - y_a)e(\gamma)$.

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Remark

Let $\gamma, \gamma' \in K^n/\langle\sigma\rangle$. The quantities $\gamma_a \bowtie \gamma'_b$ for $\bowtie \in \{=, \neq, \to\}$ are well defined if and only if $\gamma = \gamma'$.

Fixed-point cyclotomic quiver Hecke subalgebra

We now assume that $\Lambda_k = \Lambda_{\sigma(k)}$ for all $k \in K$.

Theorem (R., 2016)

The automorphism σ induces an automorphism of $\mathrm{R}_n^{\boldsymbol{\Lambda}}(\Gamma)$. Moreover:

$$\mathrm{R}_n^{\boldsymbol{\Lambda}}(\Gamma)^\sigma \simeq \mathrm{R}_n(\Gamma)^\sigma \Big/ \left\langle y_1^{\Lambda_{\gamma_1}} e(\gamma) = 0 : \gamma \in K^n/\langle\sigma\rangle \right\rangle .$$

Idea of the proof.

We construct two maps inverse to each other.

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- We define $g : \mathrm{R}_n^{\mathbf{\Lambda}}(\Gamma)^\sigma \to \mathrm{R}_n(\Gamma)^\sigma / \langle \cdot \rangle$ using the map $\mu : \mathrm{R}_n^{\mathbf{\Lambda}}(\Gamma) \to \mathrm{R}_n^{\mathbf{\Lambda}}(\Gamma)^\sigma$ defined by:

$$\mu := \frac{1}{p} \sum_{j=0}^{p-1} \sigma^j .$$

□

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1. Graded isomorphism theorem
2. Presentation of some fixed-point cyclotomic quiver Hecke subalgebras
3. Application to the case of the Hecke algebra of $G(r, p, n)$

Analogue of σ in R_n^{Λ}

We have the following situation :

$$\begin{array}{ccc} \mathrm{H}_n^{\Lambda}(q) & \overset{\sim}{\longleftrightarrow} & \mathrm{R}_n^{\Lambda}(\Gamma_{e,p}) \\ \circlearrowleft & & \circlearrowleft \\ \sigma & & ? \end{array}$$

where $\Gamma_{e,p}$ is defined by:

- its vertex set $\mathbb{Z}/e\mathbb{Z} \times \mathbb{Z}/p'\mathbb{Z} \simeq \{\zeta^j q^i : i \in \mathbb{Z}/e\mathbb{Z}, j \in \mathbb{Z}/p\mathbb{Z}\}$;
- its arrows $v \to qv$ where $v = \zeta^j q^i$.

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- its arrows $v \to qv$ where $v = \zeta^j q^i$.

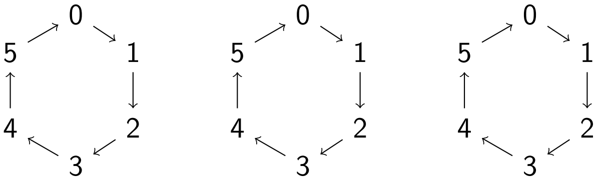
Definition

We define the map σ on $\mathbb{Z}/e\mathbb{Z} \times \mathbb{Z}/p'\mathbb{Z}$ by $\sigma(v) := \zeta v$.

Note that this map respects the arrows $v \to qv$.

Illustration

We consider the case $e := 6, p := 9$ and $p' = \frac{9}{\gcd(9,6)} = 3$. The map σ defined on $\mathbb{Z}/e\mathbb{Z} \times \mathbb{Z}/p'\mathbb{Z}$ is given by:



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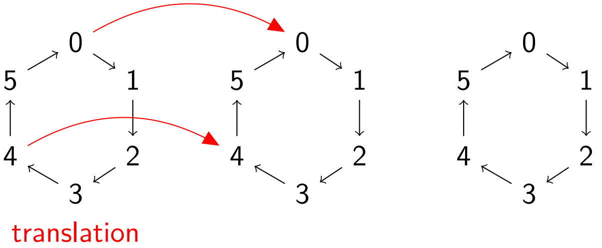

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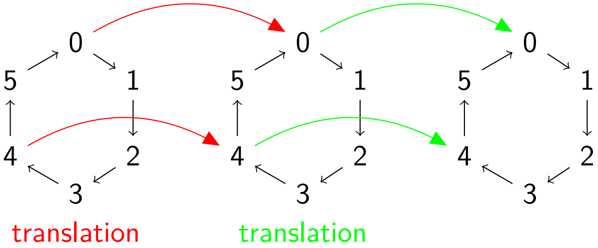

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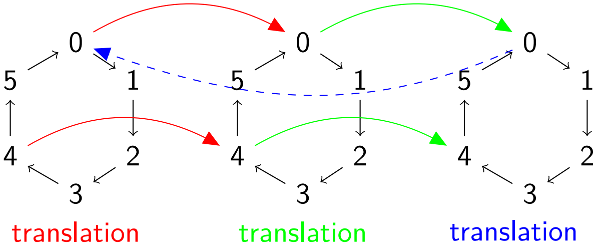

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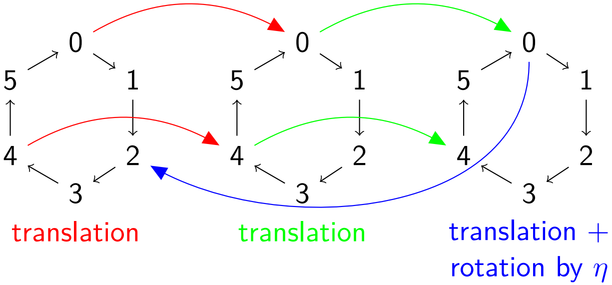

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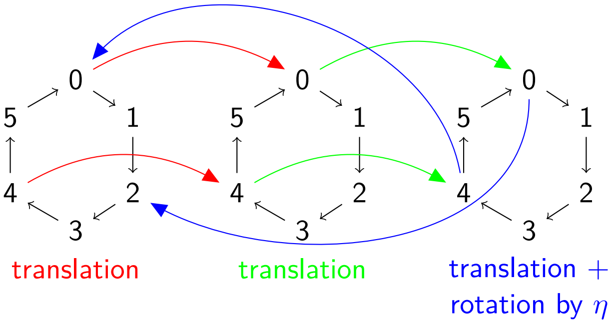

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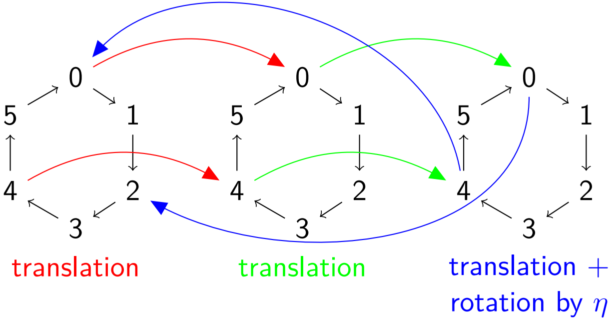

where $\eta \in \mathbb{Z}/e\mathbb{Z}$ is determined by the equality $\zeta^{p'} = q^{\eta}$.

This is a good analogue

Recall that from the map $\sigma : \mathbb{Z}/e\mathbb{Z} \times \mathbb{Z}/p'\mathbb{Z} \to \mathbb{Z}/e\mathbb{Z} \times \mathbb{Z}/p'\mathbb{Z}$ we can deduce an automorphism σ of $\mathrm{R}_n^{\boldsymbol{\Lambda}}(\Gamma_{e,p})$, defined by:

$$\begin{aligned} \sigma(e(\boldsymbol{i},\boldsymbol{j})) &\coloneqq e(\sigma(\boldsymbol{i},\boldsymbol{j})), \\ \sigma(y_a) &\coloneqq y_a, \\ \sigma(\psi_a) &\coloneqq \psi_a. \end{aligned}$$

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Theorem (R., 2016)

We can choose the elements $P_a(\boldsymbol{i}, \boldsymbol{j})$ and $Q_a(\boldsymbol{i}, \boldsymbol{j})$ such that the two maps σ coincide under the isomorphism $\mathrm{H}_n^{\mathbf{\Lambda}}(q) \simeq \mathrm{R}_n^{\mathbf{\Lambda}}(\Gamma_{e,p})$.

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Corollary

$$\mathrm{H}_{p,n}^{\Lambda}(q) \simeq \mathrm{R}_n^{\Lambda}(\Gamma_{e,p})^{\sigma}.$$

Consequences

Remark

- This choice of elements $P_a(\boldsymbol{i}, \boldsymbol{j})$ and $Q_a(\boldsymbol{i}, \boldsymbol{j})$ was used by Stroppel and Webster while studying cyclotomic quiver Schur algebras.
- The two maps σ do *not* coincide if we make the choice of $P_a(\boldsymbol{i}, \boldsymbol{j})$ and $Q_a(\boldsymbol{i}, \boldsymbol{j})$ proposed by Brundan and Kleshchev.

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Recall that we wanted to answer the following questions:

- is $\mathrm{H}^{\boldsymbol{\Lambda}}_{p,n}(q)$ isomorphic to something like a cyclotomic quiver Hecke algebra;
- is it a graded subalgebra;
- is it independent on the choosen roots of unity.

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- The two maps σ do *not* coincide if we make the choice of $P_a(\boldsymbol{i},\boldsymbol{j})$ and $Q_a(\boldsymbol{i},\boldsymbol{j})$ proposed by Brundan and Kleshchev.

Recall that we wanted to answer the following questions:

- is $\mathrm{H}^{\boldsymbol{\Lambda}}_{p,n}(q)$ isomorphic to something like a cyclotomic quiver Hecke algebra; ✓
- is it a graded subalgebra; ✓
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Consequences

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Recall that $\eta \in \mathbb{Z}/e\mathbb{Z}$ is defined by $q^\eta = \zeta^{p'}$, where p' is the smallest integer $m \geq 1$ such that $\zeta^m \in \langle q \rangle$.

Lemma

If $\widetilde{q}$ is another primitive eth root of unity, there exists a primitive pth root of unity $\widetilde{\zeta}$ such that $\widetilde{q}^\eta = \widetilde{\zeta}^{p'}$.

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